Optimization of maintenance actions in train operating companies - Fertagus case study

Marie Méchain
marie.mechain@tecnico.ulisboa.pt

Instituto Superior Técnico, Universidade de Lisboa – Portugal
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Abstract. Nowadays, efficient transport throughout Europe and the world has become a prerequisite for both freight and passenger travels. Railway transport still has to improve in EU in order to win market share from roads and sea in the future, but it is already an important mean of transport all over Europe. In Lisbon metropolitan area, Fertagus was the first private train operating company. This train operating company is running a line between Roma-Areeiro and Setubal and has its own maintenance yard. Therefore, optimizing maintenance costs is one of the main objectives of Fertagus train operating company. This work presents a mathematical model (which is a mixed integer linear programming model) that was implemented in FICO Xpress software. The model was validated and illustrated with a small-scale example. This mathematical model gives optimal technical planning as an output which reduces the cost of preventive maintenance. Real data was collected during meetings at Fertagus maintenance yard and is used in this work to obtain the minimal costs possible for preventive maintenance. Some sensitivity analysis is performed on some parameters of the mathematical model.

Keywords: Maintenance optimization, Train operating companies**,** Mixed Integer Linear Programming.

1. Introduction

In transportation companies, maintenance has a critical impact on both safety and availability. Indeed, it can be understood that if a vehicle is not maintained at all, components would fail more and more throughout use. This is of course not advisable as it would reduce availability of the fleet and could lead to critical safety issues. Therefore, companies have understood that even if maintenance costs could be quite high, it would guarantee fleet's availability and would prevent accidents. Both of these factors have a major impact on the corporate's image which is something that should be taken into consideration. In order to ensure safety, vehicles constructors require that preventive maintenance is performed within deadlines.
When it comes to maintenance, there are two ways to proceed: it can either be done when the maintenance deadline is reached or when a failure occurs. The first kind of maintenance is called ***preventive maintenance***, while the second is named ***corrective maintenance.*** So that safety is ensured, preventive maintenance interval must be optimized in order to keep the failure rate of the vehicle components under a satisfactory level. Of course, it could be tempting to perform preventive maintenance at very small intervals in order to have very low failure rates. However, if unnecessary preventive actions are performed too often, maintenance costs would dramatically increase and, moreover, early maintenance can sometimes trigger component failure. Thereby, preventive maintenance intervals should be chosen wisely by taking into account these two factors. As a result, preventive maintenance can then be optimized in order to get the cheapest maintenance costs possible that still fulfil every deadline of the company vehicles. Corrective maintenance on the other hand has a random nature and is hard to predict and thus hard to optimize.

2. Related Literature

Haghani and Shafahi (2002) studied a way to perform buses' maintenance mostly during their idle time in order to reduce the number of maintenance hours for vehicles that are pulled out of their service for inspection. The solution of the optimization program is a maintenance schedule for each bus due for inspection as well as the minimum number of maintenance lines that should be allocated for each type of inspection over the scheduled period.

Maróti and Kroon (2007) focused on finding a way to allocate to daily service a train that is due for maintenance in a maintenance yard far from the train current location. The objective is to maximize the journeys with passenger on board for a train that is due for upkeep. Indeed, if the train goes as an empty train to the next maintenance, it would significantly increase the cost of the maintenance activity. In order to solve this problem, the authors suggest using an interchange model which modifies the current plan by replacing the regular transitions by combinations of interchanges between the former tasks. Of course, the point is to lead each urgent train unit to maintenance within the deadline.

Technical planning has been studied by Doganay and Bohlin (2010) and their model has been extended by Bohlin and Wärja (2015). In this kind of planning, the time unit is a week as it is not relevant to have a detailed schedule on more than two weeks ahead of the current date. However, knowing how many trains would be maintained on a given week is valuable information. Indeed, it is useful to verify that not too many trains are under maintenance on a given week or that enough spare parts are available to perform the task. It was shown that taking spare parts into account leads to better cost savings, as it removes conflicts caused by too many trains requiring the same spare part at the same time. In Bohlin and Wärja work, inclusions in the maintenance tasks are added, i.e. if a task is included in another, there is no need to perform both in a row.

Bazargan (2015) studied how to minimize the cost of maintenance and maximize aircraft availability and then compared with several possible planning: closest to maintenance; furthest to maintenance; random maintenance; cheapest next maintenance; equal aircraft utilization. This was a study for a flight training school, and it is interesting to notice that they selected the planning with the smallest number of maintenance activities even if it was not the cheapest one. It was interesting to realize that companies would often select user-friendly solutions over solutions harder to implement even if they are more optimized.

Lai, Wang, and Huang (2017) improved the efficiency of rolling stock usage and automate the planning process. The planners are currently doing it manually with a horizon of two days which can lead to myopic decisions, far from the optimum plan. The main target of the objective function is to minimize the optimality gap between the current mileage of the train set and the upper limit each day in order to find the best planning possible.

3. Mathematical model definition

3.1 Indexes

u	train unit
t	time unit
i	maintenance activity
p	spare part
l	maintenance line

3.2 Sets

U	set of train units u
I	set of maintenance activities i
T	set of time units t
P	set of spare parts p
L_i	set of available maintenance lines l in the maintenance yard for maintenance activity i

3.3Parameters

MA_cost_i	cost of maintenance activity i
T_i	period of maintenance activity i (in time unit)
Δ_i	amount of work required to perform maintenance activity i (in man-hour)
$duration_i$	duration of the maintenance activity i. (Note; tis calculated as the ratio between Δ_i and the number of men needed to perform the maintenance activity i)
SP_cost_p	cost of having a spare part p per time unit t
κ_{ip}	number of spare parts p needed to perform maintenance activity i
R_p	duration of the maintenance of spare part p (in time unit)
A_p	maximum amount of spare parts p
O_{ui}	time interval between last maintenance activity i and beginning of planning horizon for train unit u

3.4 Constants

H	planning horizon
S	shunting cost
k	maximum working load per time unit t (in man hours)
max_time	maximum working time per time unit t (in hours)
N	number of maintenance activities I (Note: it is the cardinality of the set I)
delay	amount of time needed to move a train from a maintenance line l (in hours)
u_1	maximum number of train units available
u_2	number of train units needed to perform daily service

The parameter k is calculated as the number of men working times the time duration of maintenance per day times the number of working days per time unit t.

The parameter max_time is calculated as the time duration of maintenance per day times the number of working days per time unit t.

3.5 Variables

x_{uitl}	binary variable set to 1 if maintenance activity i is performed on train unit u at t time unit, and set to 0 otherwise.
y_{ut}	binary variable set to 1 if unit u is under maintenance at t time unit, and set to 0 otherwise.
U_p	non-negative integer variable corresponding to the minimum

amount of spare part required to perform the technical planning

3.6 Objective function

$$minimize \ \sum_{u\in U}\sum_{i\in I}\sum_{t\in T}\sum_{l\in L_i} \text{MA\_cost}_i * x_{uitl} + \sum_{u\in U}\sum_{t\in T} S * y_{ut} + H * \sum_{p\in P} \text{SP\_cost}_p * U_p + \frac{1}{(u_1-u_2)*N*H}\sum_{u\in U}\sum_{i\in I}\sum_{t\in T}\sum_{l\in L_i}(H-t) * x_{uitl} \quad (1)$$

Subject to:

$$\sum_{j=t}^{t+T_i}\sum_{l\in L_i} x_{uitl} \geq 1 \quad \forall u \in U, i\in I, t\in \{1,\dots,H-T_i+1\} \quad (2)$$

$$\sum_{j=1}^{T_i-O_{ui}}\sum_{l\in L_i} x_{uitl} \geq 1 \quad \forall u \in U, i \in I \ such\ that\ T_i - O_{ui} \leq H \quad (3)$$

$$y_{ut} \geq x_{uitl} \quad \forall u \in U, i\in I, t\in T, l\ in\ L_i \quad (4)$$

$$\sum_{u\in U}\sum_{i\in I}\sum_{l\in L_i}\sum_{j=t}^{t+R_p} \kappa_{ip} * x_{uitl} \leq U_p \quad \forall p\in P, t\in\{1,\dots,H-R_p\} \quad (5)$$

$$U_p \leq A_p \quad \forall p\in P \quad (6)$$

$$\sum_{u\in U}\sum_{i\in I}\Delta_i * x_{uitl} \leq k \quad \forall t\in T, l\in L_i \quad (7)$$

$$\sum_{u\in U}\sum_{i\in I} duration_i * x_{uitl} + delay * \left(\sum_{u\in U}\sum_{i\in I} x_{uitl} - 1\right) \leq \text{max\_time} \quad \forall t\in T, l\ in\ L_i \quad (8)$$

$$\sum_{l\ in\ L_i} x_{uitl} \leq 1 \quad \forall u\in U, \ \forall i\in I, \ \forall t\in T \quad (9)$$

x_{uitl} is binary $\forall u \in U, i\in I, t\in T, l\ in\ L_i$ (10)

y_{ut} is binary $\forall u\in U, t\in T$ (11)

U_p is non-negative integer $\forall p\in P$ (12)

The objective function (1) is the total cost of preventive maintenance over a year. It was adapted from an objective function found in a paper by Doganay and Bohlin (2010) that fitted the objective of minimizing all costs of maintenance of trains in a railway maintenance yard. This function is composed of four different cost components which are the maintenance cost - denoted A; the shunting cost - denoted B; the spare parts cost - denoted C and finally a cost to avoid early maintenance - denoted D. The objective function is then A+B+C+D. All the cost components are explained in detail in the following subsections.

The maintenance cost A is the cost of doing every maintenance task over the planning horizon. Each maintenance costs MA_cost_i were given previously as an input; they correspond to the cost of doing a specific maintenance task i. The cost component A can be expressed as the sum of all the maintenance costs of the maintenance activities performed on every trains, every line and at every time period until the horizon.

The cost component B is the shunting cost; it corresponds to the cost of pulling a train out of its regular duty in order to perform maintenance on this train. It can be expressed as the sum of the shunting cost per week of all trains stopped every week of the planning horizon.

The cost component C is the cost of keeping spare parts that need to be kept in good conditions even when they are not used. The spare part cost is also defined previously by the user; it is commonly estimated as a percentage of the initial price of the spare part. The cost component C can be set as the product of the duration in time units of the planning horizon and the sum of the spare part cost times the amount of each spare part. It must be highlighted that in this model, minimum amount of spare parts remains the same throughout the year. Therefore, U_p is chosen so that it would fulfil all maintenance activities on all trains and all time period over the planning horizon.

The last cost component is a term to discourage early maintenance as it is both costly and likely to trigger some early failure of the components. The cost component D can be seen as a penalty if the last preventive maintenance before the end of planning horizon is performed too early. It is the product of $\frac{1}{(u_1-u_2)*N*H}$ which is a weighted penalty, times the distance between the last maintenance perfomed and the end of the planning horizon $(H-t) * x_{uitl}$. The closer to the end of planning horizon the maintenance activity is performed the smaller the penalty cost. The weighted penalty is made of the inverse of the product of the total number of maintenance activities, multiplied by the planning horizon times the number of spare trains; i.e. the difference between the number of train units owned by the train operating company and the usefull number of trains to perform daily service.

Constraint (2) is imposed in order to have each maintenance task i occurring at least once every period T_i for all train units, maintenance tasks and time periods.

Constraint (3) states that every maintenance task i which is due by the end of the planning horizon H is performed at least once.

Constraint (4) imposes that if x_{uitl} is equal to one, i.e. if maintenance activity i is scheduled in a particular time period t for train unit u and line l, then y_{ut} must be equal to one. Therefore, every shunting must be taken into account.

Constraint (5) requires that the number of spare parts needed is greater than the greatest number in service at any single occasion.

Constraint (6) bounds the number of spare part in order to stay under the limit chose by the user. This upper bound represents the maintenance yard's storage capacity.

Constraint (7) limits the total working load performed during a week under the maximum amount of work that can be done within one time unit. In this model, the maximum amount of work is not time dependent, which might be changed if the simplification is not relevant.

Constraint (8) makes the maintenance duration on each line stays under the maximum amount of working time per time unit t (time per day times number of working days). A delay, corresponding to the time required to move the trains is added. This delay is multiplied by the total number of movement $\sum_{u \in U} \sum_{i \in I} x_{uitl} - 1$ which is the total number of maintenance activities performed on all the trains minus one; which also is the number of movements on a given maintenance line l.

Constraint (9) imposes that, for each maintenance activity i of train u at a given time t is either not performed (left hand side equal to zero) or performed in a given maintenance line (left hand side equal to 1). The same maintenance activity i on the same train u can only be performed in one maintenance line l.

Constraint (10) makes x_{uitl} a binary variable for all train unit, maintenance activity, time unit and maintenance line l

Constraint (11) makes y_{ut} a binary variable for all train and time units

Constraint (12) imposes that U_p is a non-negative integer for all spare parts

4. Case study of Fertagus

In part 4, Fertagus train operating company is presented briefly and problem specifications are introduced. The parameters of mathematical model are displayed in a table with all values given in monetary units for the sake of confidentiality.

4.1 Fertagus, a train operating company

Fertagus trains run on a line of 54 kilometres that crosses the "*25 de Abril"* bridge; and stop at 14 stations. Total travel duration between Roma-Areeiro and Setúbal is 57 minutes and the bridge crossing is only 7 minutes long.

Fertagus is a train operating company whose name derives both from "*caminhos-de-ferro*" and the Tagus river's name. It became the first private rail operator in Portugal when it won the call for bids for the line between Lisbon's city centre and the Setubal district area. As a result, it now has a contract that ensures availability, cost and duration of travels. As it was the first private operator of Lisbon metropolitan area, it is interesting to notice that availability is part of the contract with Lisbon's city centre. Indeed, in order to optimize maintenance, it must be kept in mind that no train can be pulled out of service to go to maintenance if there is no backup train available. In order to guarantee that this would not happen, Fertagus chose to have eighteen trains when only seventeen are necessary to perform the current operation schedules. The question of whether or not this could be done differently is out of the scope of the present work and is left for further research. However, it might not be necessary to have an additional train if instead of doing mileage-based maintenance, Fertagus was using a condition-based maintenance approach.

It must be said that as Fertagus does not own the railway line, infrastructure charges must be paid to the infrastructure manager, IP, and the railway infrastructure maintenance is not up to Fertagus. This also means that Fertagus trains are not the only trains running over the line between Roma-Areeiro and Setubal, which might result in technical issues as not all trains have the same requirements. Indeed, during one of our visits to the maintenance yard, Eng. João Duarte told us that since the line started to be used by other trains going faster, unusual wear was noticed on all the trains' wheelsets.

Three meetings with Fertagus staff were scheduled and some additional information were given by email and phone calls. This work benefited from two main contacts, Engineer João Grossinho and Engineer João Duarte in the maintenance yard of Fertagus.

Fertagus maintenance yard comprises 10 lines. Although they are numbered from 1 to 12, lines number 3 and 4 were never built but were designed in the original maintenance yard's plans. Out of these lines, only 3 are used to perform maintenance tasks, respectively 10, 11 and 12. The other lines are used as testing or parking lines. A summary of all lines exploitation can be found in Table 1 below.

Table 1: Lines of Fertagus maintenance yard

Line number	**Use in the maintenance yard**
1	Several tests
2	Several tests
5	Parking
6	Parking
7	Parking
8	Parking
9	Cleaning operations & conservation cleaning
10	Maintenance with catenary
11	Maintenance with catenary
12	Maintenance without catenary (includes pantograph replacement)

4.2 Specific input parameters for MILP formulation

In order to model the case study, some information about the maintenance activities were gathered in order to have the correct inputs for the parameters of the mathematical model. During meetings, Fertagus maintenance activities were explained by Eng. João Grossinho and Eng. João Duarte and summarized on the tables below. While the first one corresponds to the maintenance activities scheduled by Fertagus maintenance crew; the second table sums up the maintenance activities scheduled by a consultancy company which provides support for major renewals. Because of the way the program was built, only the maintenance activities scheduled by Fertagus maintenance crew could be taken into account. The integration of R1, R2 and R3 major renewals in the mathematical model is left for further research.

Table 2: Maintenance tasks which are not performed by Fertagus crew

Maintenance activity	**Tasks performed during MA**	**Period**	**Time needed to perform MA**	**Men force required**	**Cost**
R3	Pantograph are replaced	Every 600,000 km	Unknown	Unknown	Unknown
R2	replacement of wheelsets, bogies are repaired and sent to EMEF or RENFE	Every 1,200,000 km	Unknown	Unknown	Unknown
R1	replacement of wheelsets, bogies are repaired and sent to EMEF or RENFE	Every 1,800,000 km	Unknown	Unknown	Unknown

Table 3: Maintenance tasks performed by Fertagus crew

Maintenance activity	**Tasks performed during MA**	**Period**	**Time needed to perform MA**	**Men force required**
ETS *"Ensaios e Trabalhos Sistemáticos"*	Mostly inspection activities	Every 5 weeks	1h30-2h00	4
VEq *"Visita de Equipamento"*	Inspection of motor block, pressure check, etc.	Every 37,500 km	6h00	4
VP *"Visita de Portas"*	Doors check-up	Every 150,000 km	6h00	4
VL *"Visita de Lubrificação"*	Lubrication check-up	Every 120,000 km	4h00-6h00	4
VEl *"Visita Eléctrica"*	Electric system check-up	Once a year (before winter)	40h00 (not continuous)	4
VS *"Visitas Sazonais"*	Biannual check-up	Twice a year (beginning of Spring and end of Summer)	12h00	4
TRF *"Torneamento dos rodados"*	Wheelsets turning	Every 120,000 km	16h00 during weekend	2
V1	Some parts of the pantograph are maintained	Every 300,000 km	Unknown	4

The mathematical model's parameters for the Fertagus case study were extracted from the two tables above and completed through additional meeting; emails or phone calls. All parameters used for the Fertagus case study can be found in the following tables (Table 4, Table 5, Table 6, Table 7, Table 8 and Table 9).

Table 4: Sets of the mathematical model

Sets	**Values**
U	{1,...,18}
I	{1,...,16}
T	{1,…,53}
P	{1,…,4}

In Table 4 the sets of Fertagus case study are displayed; they are eighteen trains so U is a set of integers going from 1 to 18. Furthermore, sixteen maintenance activities can be performed in Fertagus maintenance yard which implies that I is a set of integers from 1 to 16. The planning horizon of the technical planning we want to achieve is a year so, since the time unit is a week, T is a set of integers from 1 to 53. Finally, four different spare parts are stored in Fertagus maintenance yard so P is a set of integers from 1 to 4.

Table 5: Parameters of the mathematical model depending on the maintenance activity i

i	MA_type_i	MA_cost_i	T_i (in weeks)	Δ_i (in man-hour)	$duration_i$	L_i
1	ETS	614,42	5	10	2,5	{10 11}
2	VEQ	1720,37	16	28	7	{10 11}
3	LUB	1018,98	53	14	3,5	{10 11}
4	POR	829,17	63	14	3,5	{10 11}
5	TRF	2522,22	63	42	21	{10 11}
6	LL	815,28	63	14	4,6	{10 11}
7	ELET1	3516,44	53	12,4	3,1	{10 11}
8	ELET2	3516,44	53	12,4	3,1	{10 11}
9	ELET3	3516,44	53	12,4	3,1	{10 11}
10	ELET4	3516,44	53	12,4	3,1	{10 11}
11	ELET5	3516,44	53	12,4	3,1	{10 11}
12	AC	793,29	53	14	7	{10 11}
13	BAT1	396,64	53	3,5	0,88	{12}
14	BAT2	396,64	53	3,5	0,88	{12}
15	MR	56,25	26	1	1	{10 11}
16	V1	2457,68	136	40	10	{12}

In Table 5 all parameters depending on the maintenance activities i are summarized. The first line includes the name of maintenance activity 1 which is ETS, its cost in monetary units which is 614,42. Then the period of the ETS maintenance is displayed in weeks and is equal to five weeks. This means that maintenance activity 1 (called ETS) is due every five weeks. Then, both the working load and the duration of the maintenance activity 1 are given. ETS maintenance is a 10 man-hours maintenance activity and lasts 2,5 hours long. Because the working load and the duration are linked by the relation "working load = duration * working men", it can be deduced that 4 men are needed to do maintenance activity 1. Finally, the set of maintenance lines where maintenance activity 1 can be performed is displayed. It can be read that maintenance ETS can be performed either on line 11 or on line 12 of Fertagus maintenance yard. Indeed, it was explained in Table 1 that line 10 and 11 are equipped with the same tools and can therefore be used for the same maintenance activities.

Table 6: Parameters of the mathematical model depending on the spare parts p

p	SP_type_p	SP_cost_p	R_p	A_p
1	wheelset	104,17	1	20
2	trailer bogie	1041,67	0	20
3	motor bogie	1041,67	1	20
4	pantograph	416,67	2	20

Parameters that depends on the spare parts p are displayed in Table 6. In the first line are given all information about the spare part 1; the first piece of information is the spare part's name which is a wheelset. Then, the cost of the spare part 1 per week is given in monetary units and is 104,17. The next parameter is the number of week needed to maintain the spare wheelset and its value is one which means that when a spare wheelset is sent to maintenance it will be unavailable for one week. Finally, the maximum number of spare part 1 is given according to the maintenance yard storage area. Because Fertagus maintenance yard is relatively large, it is assumed that storage would not be an issue; this is why the maximum number of wheelset is set to 20. Here the largest number of spare parts does not constraint the solution, it only reduces the number of values that the solver will test. Thus, it reduces the computational time.

Table 7: O_{ui} parameter of the mathematical model

Train number\MA	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	4	3	19	50	46	25	40	40	40	40	40	13	16	16	19	45
2	2	5	26	7	42	25	40	40	40	40	40	13	16	16	9	40
3	4	0	37	19	20	24	36	36	36	36	36	12	16	16	16	54
4	3	10	40	30	15	20	35	35	35	35	35	12	16	16	18	56
5	3	15	16	16	15	24	15	15	15	15	15	12	15	15	24	56
6	0	14	2	42	36	23	14	14	14	14	14	11	15	15	3	54
7	3	15	18	23	49	23	13	13	13	13	13	11	15	15	17	47
8	0	1	30	30	51	22	12	12	12	12	12	10	14	14	4	55
9	1	14	38	14	47	22	12	12	12	12	12	10	14	14	11	46
10	1	0	5	37	39	21	11	11	11	11	11	9	14	14	0	35
11	3	9	43	16	44	21	10	10	10	10	10	9	13	13	5	41
12	4	11	52	45	43	20	7	7	7	7	7	8	13	13	6	35
13	2	10	17	3	45	20	6	6	6	6	6	8	13	13	18	41
14	2	14	13	48	46	19	5	5	5	5	5	7	12	12	16	35
15	4	6	33	22	40	19	4	4	4	4	4	7	12	12	8	49
16	0	1	46	45	36	18	3	3	3	3	3	6	12	12	23	37
17	3	15	39	55	48	18	2	2	2	2	2	6	11	11	20	43
18	4	12	28	35	35	17	1	1	1	1	1	5	11	11	14	44

Initial conditions about Fertagus trains can be found in Table 7. The different maintenance activities are put in the columns while the different lines correspond to different trains. The first line set all the initial conditions for train 1. The first value is the distance between the last maintenance activity 1 and the beginning of the planning horizon and is 4; which means that the last maintenance activity 1 (called ETS) was performed on train 1 four weeks ago. It must be highlighted that all values of O_{ui} can be deduced from this table. For example, O_{23}, which corresponds to the last time maintenance activity 3 was performed on train 2, is set to 23 weeks in the above table. Therefore, maintenance activity 3 was done 23 weeks before the beginning of the planning horizon.

Table 8: κ_{ip} parameter of the mathematical model

SP\MA	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 8 summarizes the values of parameter κ_{ip} which is the amount of spare part p required to perform maintenance activity i. Sixteen different maintenance activities can be done in Fertagus maintenance yard; but they do not all require the same spare parts. Most of Fertagus maintenance activities, such as maintenance activity 1, do not need any spare part. This can be seen on the first column of the above table, where all values are set to zero. It must be said that spare parts are mostly used during corrective maintenance in Fertagus case study which is why the cost of preventive maintenance is lower. However, some maintenance activities of

Fertagus railway operating company still need spare part to be done. It is the case of maintenance activity 5 that requires one spare part one (which is a wheelset) to be performed.

Table 9: Constants of the mathematical model

Constants	Values
H	53
S	5000
k	160
max_time	40
N	17
delay	0.16
$\mathbf{u_1}$	18
$\mathbf{u_2}$	17

All constants of the mathematical model can be found in Table 9. First the planning horizon H whose value is 53 weeks. The shunting cost is then set to 5000 monetary units, this value was initially given as an approximation and will thus be the subject of a sensitivity analysis. The maximal working load k which is 160 man-hours is calculated as the product of the number of men working in Fertagus maintenance yard by the number of working hours per day times the number of useful days of the week. In the case study it is then, 4 men * 8 hours * 5 (days) = 160 man-hours. The maximal working time per week is 40 hours and is calculated as the product of the number of working hours per day times the number of useful days in a week. In Fertagus case study it is 8 hours * 5 (days) = 40 hours. It is interesting to realize that the maximal working load and the maximal working time are related to one another with the equation working load = working time * number of men.

5. Results

In chapter 5 several studies can be found. Firstly, an analysis of optimality gap over calculation time; the optimality gap mentioned above, is calculated as the percentage of the ratio of the difference between the value of the objective function and the lower bound and the value of the objective function. In the next subsections, a sensitivity study of both the shunting cost component and the maximal working time per week. It should be made clear that the shunting cost is the cost related to moving trains to the maintenance yard.

5.1 Analysis of optimality gap as a function of calculation time

As it was said in section 3, when the size of the problem increases so does the computational time to get to the optimal solution. Indeed, if optimality of the solution is not taken in consideration, then a feasible solution can always be found within few minutes. However, if optimality of the solution is important, the optimality gap corresponding to the feasible solution is an indicator of the optimality of the solution. The closer the optimality gap is to zero, the better the solution is. When the size of the problem is relatively small (such as 752 variables), the optimal solution can be found in a tenth of a second as it was the case in the illustrative example. Nevertheless, in the case of Fertagus the computational time is much larger, due to a significant increase of the size of the sets. It is then interesting to study the evolution of the optimality gap with respect to computational time in order to know when the solution can be considered as satisfying. Indeed, most of the time the calculus is stopped before the exact solution is reached (i.e. when the optimality gap is 0) in order to save computational time.

In this analysis, the computational time was increased from 1 minute to 24 hours, and the graph of optimality gap versus computational time can be found in Figure 1. The goal of this study is to be able to select the smallest computational time whose corresponding optimality gap is acceptable. It can be seen on the graph that after a calculation time of one hour, a optimality gap around 0,6% is achieved. When computational time is increased to one day, the optimality gap becomes slightly less than 0,5% which is better but may not worth the additional time spent to minimize the cost. Consequently, the computational time was chosen to be set to one hour for all further analysis in this chapter.

The detailed values of both the computational time and the associated optimality gap can be found in Figure 1 and Table .

Table 10: Values of computational time and corresponding optimality gap

Computational time (s)	Optimality gap (%)
61 (1 min)	2,67
304 (5 min)	2,67
601,4 (10 min)	2,30
913,5 (15 min)	1,55
1798,5 (30 min)	0,97
2701,2 (45 min)	0,97
3602,6 (1 h)	0,63
5411,9 (1h30)	0,63
18014,6 (5 h)	0,63

36006,9 (10 h)	0,63
86405 (24 h)	0,60

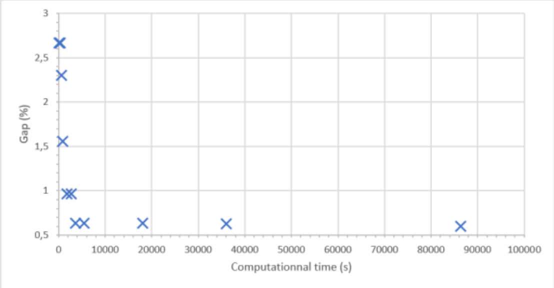

Figure 1: Graph of optimality gap with respect to computational time

In can be seen in Figure 1 that the variation of the optimality gap is a lot faster at the beginning. Indeed, during the first hour, the optimality gap decreases from 2,67% to 0,63% while in the remaining twenty-three hours it only decreases from 0,63% to 0,6%. This graph proves that the longer the computational time, the smaller the optimality gap but it must be highlighted that the evolution is not linear.

Analysis of maintenance costs as a function of shunting cost component

In Fertagus case, the shunting cost component value is set to 5000 monetary units. However, this cost was provided as an approximated value so it was worth doing a sensitivity analysis on this parameter of the mathematical model. In order to study the influence of the parameter on the total cost of the maintenance, the shunting cost component is varied from 4500 monetary units to 5500 monetary units. This corresponds to an increase and a decrease by 10% of this cost component.

The graph of the total maintenance cost with respect to the shunting cost component can be find in Figure 2. This graph is however not easy to analyse which is why another graph can be found in Figure 3 representing the variations in percentage of the total maintenance cost induced by the variations in percentage of shunting cost component.

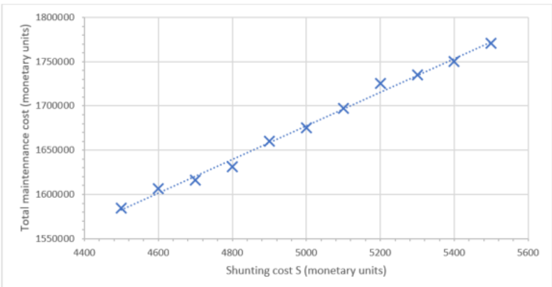

Figure 2: Total maintenance cost versus shunting cost component

It can be seen in Figure 2 that the evolution of the total preventive maintenance cost is similar to a linear evolution between 4500 and 5500 monetary units. The linear curve displayed on the above figure has an equation that is y =18985x + 728178; this linear function values have an average difference of 0,2% with the real values.

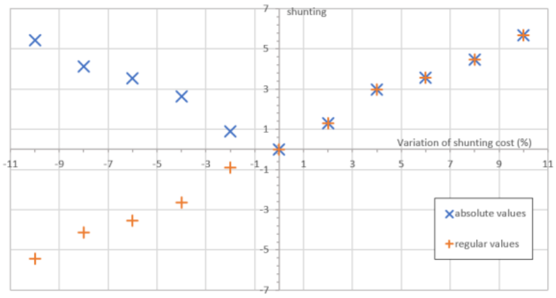

Figure 3: Variation of total cost with respect to shunting cost component variations

Two different curves can be found in Figure 3, one is called "regular values and the other is "absolute values". The first one, with "regular values", corresponds to the variation in percentage of the total maintenance cost with respect to the variation in percentage of shunting cost (between the reference value 5000 and the value used in the mathematical model). The other curve the "absolute values" corresponds to the absolute value of the variation in percent of the total maintenance cost with respect to the variation in percentage of shunting cost.

A variation of shunting cost of 2% induces a variation of total maintenance variation of 1,3% which corresponds to a relation of 0,65 between the total maintenance variation and the shunting cost variation. In Table a summary of the relations between shunting cost component and total maintenance variations. The average value of these relations is 0,58 which means that the variation of shunting cost component induces a smaller variation of the total maintenance cost.

It is interesting to notice that the "absolute values" curve in *Figure 3: Variation of total cost with respect to shunting cost component variations*Figure 3 is symmetric with respect to the y-axis. It means that an increase of 3% of the shunting cost component will induce a raise on the total cost; and this raise has the same absolute value than the diminution of the total costs induced by a decrease of 3% in the shunting cost component.

Table 11: Relation between the two variations

Shunting variation (%)	Total maintenance variation (%)	Relation between the two variations
-10	-5,43	0,54
-8	-4,12	0,51
-6	-3,54	0,59
-4	-2,63	0,66
-2	-0,90	0,45
0	0,00	Not relevant

2	1,30	0,65
4	2,97	0,74
6	3,58	0,60
8	4,46	0,56
10	5,68	0,57

The relation between the shunting variation in percentage and the total maintenance variation in percentage can be found in Table 11. The value for a variation of zero percent is not considered to be relevant as a division by zero would be involved otherwise. It is interesting to highlight that all the ratio

Analysis of total maintenance costs as a function of working time per week

Fertagus preventive maintenance is done in a shift of 8 hours per day, and five days a week; which corresponds to a current maximum working time in the maintenance yard of 160 hours/week. In order to quantify the impact of the time allocated to preventive maintenance, a study is performed to see the evolution of total maintenance costs with respect to the evolution of the maximum working time. It must be highlighted that even if only one value changes, it affects two parameters of the mathematical model which are k and max_time.

The graphs on Figure 4 and Figure 5 show the variations of the total maintenance costs as a function of the variation of the time allocated to preventive maintenance per week. The time allocated varies from 36 hours to 44 hours; 40 being the reference (all values can be found in Table). When the time allocated is 36 hours, there is no value for the optimality gap nor for the total maintenance costs because no feasible solution could be found. On the contrary when the time allocated is 44 hours, the optimal solution is find in 2445,8 seconds (40min and 45 seconds) which means the calculus stops before the end of the reference computational time. For an allocated time of 42 hours the solution is optimal after 2684 seconds (44 min and 46 seconds). The reference for the computational time was found in the first subsection of this chapter and was set to one hour.

Table 12: Values of optimality gap and total maintenance costs for different time duration

allocated man-hours per week	allocated hours per week	optimality gap after one hour (%)	total maintenance cost (monetary units)
36	144	-	No feasible solution
38	152	1,84	1701030
40	160	0,63	1675360
42	168	0	1659800
44	176	0	1659750

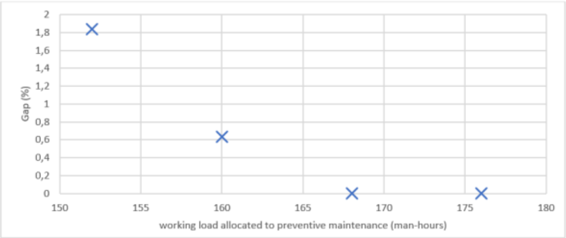

Figure 4: Graph of optimality gap versus time allocated per week

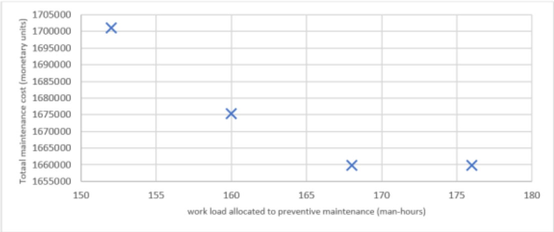

Figure 5: Graph of total maintenance costs versus time allocated per week

The horizontal axis of Figure 4 and Figure 5 are the same which is logical considering that the working load allocated and the working time allocated are related through the equation working:
load = working time * number of men. During the sensitivity analysis, the number of men do not change which explains why only one parameter was used to plot the graph of the sensitivity analysis.

On Figure 4 it can be noticed that the optimality gap value is drastically increased when the working load is lower than 160 man-hours. If the working load is lower 152 man-hours, there is even no feasible solution that can be found. There are two possible explanations for this; either 160 man-hours working load is optimal in Fertagus case study; or the initial conditions have a lot of influence on the technical planning. Indeed, a maintenance which is done by four working men for several planning horizons could lead to initial conditions that require a preventive maintenance done by four men. Therefore, after having performed preventive maintenance a certain way for a long time it could be difficult to change the way to do things.

6. Conclusion and future research

In the final part, the conclusion of the performed research can be found as well as some limitations and possible further step of the future research.

6.1 Conclusion

Optimizing total costs of preventive maintenance is of course the objective of every companies since it would have a non-neglectable effect on the budget. The goal of this thesis was to create a mathematical model that would provide an optimal technical planning reducing the total preventive maintenance costs to a minimum. The mathematical model created was adapted to the specific case of Fertagus railway operator but can very easily be modified to fit to any company's specifications.

One of the objectives of this thesis was to prove that this program would give the optimal feasible technical planning if at least one can be found. It is important to underline that whenever a parameter was too restrictive, the mathematical model would say that no feasible solution could be found. Several simulations were made in order to quantify the sensitivity of some parameters and the mathematical model was considered as satisfying.

Moreover, it is interesting to realize that the mathematical model is able to give a feasible solution with a optimality gap of less than one percent in less than one hour. Indeed, most company would care about the optimality of the solution. This is why, even if a technical planning is performed once a year, it is still compelling that the computational time for a company size calculus stays relatively low.

6.2 Limitations

This mathematical model enables to find an optimal technical planning but it is of course user input dependent and this is a major limitation. Indeed, if the user inputs do not represent correctly the real-life situation, the technical planning could hardly be optimal, even if an optimal solution is found. This is the case in Fertagus maintenance yard, which means that the solution found are actually not the best possible. Indeed, some maintenance activities are performed by Fertagus maintenance crew, but some others (such as R1, R2 and R3) are performed by a consultant company. These maintenance activities were not taken into consideration and consequently the total maintenance cost find by the program cannot be the actual optimal one.

6.3 Future Research

As said in the previous subsection, all maintenance activities performed by a consultancy company are not taken into account. In order to provide a total maintenance costs which would be more accurate, a new cost component will have to be added. This cost component should reflect the cost of having these maintenance activities done outside of Fertagus maintenance yard. It should be made of both a cost of unavailability and a labour cost and will have to be added to the mathematical model.

Moreover, even if this thesis provides a first approach of an optimized technical planning, it must be kept in mind that the cost of preventive maintenance represents barely a fifth of the costs of corrective maintenance. This means that if a company really wants to save costs on maintenance, corrective maintenance should really be taken into considerations. In Fertagus case, corrective maintenance is done every day during eight hours a day, right after the eight hours dedicated to preventive maintenance. One way of taking into account corrective maintenance would be to add a corrective maintenance cost component in the mathematical model. However, ideally a new program should be created since corrective maintenance can only be predicted through statistics while preventive maintenance has fixed period.

Finally, it must be said that obtaining an optimal technical planning is only the first step. After this, it must be verified that the optimal technical planning is feasible thanks to an operational planning. Indeed, some of the constraints of the maintenance yard cannot be implemented in the technical planning mathematical model. This highlights the fact that a new mathematical model, that would take the optimal technical planning as an input, should be created. Indeed, with both an optimal technical planning and an optimal operational planning found, the total maintenance costs of a company would be significantly improved.

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